Problems Based on Removing x From the integrand.

1. Evaluate $\displaystyle\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$

Solu^n:

$$I = \int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x} \quad -(1)$$

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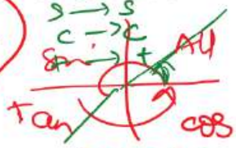
By property: $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$\pi, 2\pi$

$s \rightarrow s$

$c \rightarrow c$



$$I = \int_0^{\pi} \frac{(\pi-x)\,dx}{a^2\cos^2(\pi-x) + b^2\sin^2(\pi-x)}$$

$\sin(\theta+\pi) = \sin$

$s(\theta+2\pi) = \cos$

$s(\theta+\pi/2) = c$

$c(\theta+3\pi/2) = s$

$$= \int_0^{\pi} \frac{(\pi-x)\,dx}{a^2\cos^2 x + b^2\sin^2 x} \quad -(2)$$

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Adding (1) + (2)

$$I = \int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x} \quad \text{—(1)} \quad ; \quad I = \int_0^{\pi} \frac{(\pi - x)\,dx}{a^2\cos^2 x + b^2\sin^2 x}$$

$$2I = \int_0^{\pi} \frac{(\cancel{x} + \pi - \cancel{x})\,dx}{a^2\cos^2 x + b^2\sin^2 x}$$

$$2I = \pi \int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$$

Prop: $\int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx$ if $f(2a - x) = f(x)$

$= 0$ if

$$\cancel{2} I = \cancel{2}\pi \int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x}$$

$$I = \pi \int_0^{\pi/2} \frac{\frac{dx}{\cos^2 x}}{a^2 + b^2 \frac{\sin^2 x}{\cos^2 x}}$$

$$= \frac{\pi}{b^2} \int_0^{\pi/2} \frac{\sec^2 x \, dx}{\frac{a^2}{b^2} + \tan^2 x}$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

Let $\tan x = t$

$$\sec^2 x\,dx = dt$$

$x \to 0,\quad \tan 0 \to 0 \Rightarrow t = 0$

$x \to \pi/2,\quad \tan \pi/2 \to \infty \Rightarrow t = \infty$

$$I = \frac{\pi}{b^2}\int_0^\infty \frac{dt}{\left(\frac{a}{b}\right)^2 + t^2} = \frac{\pi}{b^2}\cdot\frac{b}{a}\left[\tan^{-1}\left(\frac{bt}{a}\right)\right]_0^\infty$$

$$= \frac{\pi}{ba}\left[\tan^{-1}(\infty) - \tan^{-1}(0)\right] = \frac{\pi}{ba}\left[\frac{\pi}{2}\right] = \frac{\pi^2}{2ab}$$