

## Problems Based on Removing $x$ From the integrand.

1. Evaluate 
$$\int_0^{\pi} \frac{x \, dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

Solu<sup>n</sup>: 
$$I = \int_0^{\pi} \frac{x \, dx}{a^2 \cos^2 x + b^2 \sin^2 x} \quad \text{--- (1)}$$

24.  $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$   
 $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + C$   
 $\int \frac{dx}{\sqrt{x^2-a^2}} = \cos^{-1} \frac{x}{a} + C$

25.  $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + C$   
 $\int \frac{dx}{\sqrt{x^2-a^2}} = \cos^{-1} \frac{x}{a} + C$

26.  $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + C$   
 $\int \frac{dx}{\sqrt{x^2-a^2}} = \cos^{-1} \frac{x}{a} + C$

27. Problem Based on Removing  $x$  From the integrand.  
 Evaluate  $\int_0^{\pi} \frac{x \, dx}{a^2 \cos^2 x + b^2 \sin^2 x}$   
 Solu<sup>n</sup>:  $I = \int_0^{\pi} \frac{x \, dx}{a^2 \cos^2 x + b^2 \sin^2 x}$  --- (1)

28.  $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + C$   
 $\int \frac{dx}{\sqrt{x^2-a^2}} = \cos^{-1} \frac{x}{a} + C$

29.  $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + C$   
 $\int \frac{dx}{\sqrt{x^2-a^2}} = \cos^{-1} \frac{x}{a} + C$

By property  $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

$I = \int_0^{\pi} \frac{(\pi-x)}{a^2 \cos^2(\pi-x) + b^2 \sin^2(\pi-x)} dx$

$= \int_0^{\pi} \frac{(\pi-x)}{a^2 \cos^2 x + b^2 \sin^2 x} dx \quad - (2)$

$\frac{\pi}{2} \rightarrow \frac{2\pi}{2}$   
 $s \rightarrow s$   
 $c \rightarrow -c$   
 $\sin \rightarrow -\cos$   
 $\cos \rightarrow \sin$   
 $\tan \rightarrow -\cot$   
 $\sin(\pi) = 0$   
 $\cos(\pi) = -1$   
 $\sin(\pi + 2n) = \sin$   
 $\cos(\pi + 2n) = -\cos$   
 $\sin(\frac{\pi}{2} + n) = \cos$   
 $\cos(\frac{\pi}{2} + n) = -\sin$

- 24.  $\int_0^a \frac{dx}{\sqrt{a^2-x^2}} = \frac{1}{a} \sin^{-1} \frac{x}{a} + C$   
 $\int_0^a \frac{dx}{\sqrt{x^2-a^2}} = \frac{1}{a} \ln \left| \frac{x}{a} + \sqrt{\frac{x^2}{a^2} - 1} \right| + C$   
 $\int_0^a \frac{dx}{\sqrt{a^2+x^2}} = \frac{1}{a} \ln \left| \frac{x}{a} + \sqrt{\frac{x^2}{a^2} + 1} \right| + C$
- 25.  $I = \int_0^{\pi} \frac{dx}{\sin x} = \int_0^{\pi} \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \int_0^{\pi} \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}}$   
 $I = \int_0^{\pi} \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \int_0^{\pi} \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}}$
- 26.  $I = \int_0^{\pi} \frac{dx}{\sin x} = \int_0^{\pi} \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \int_0^{\pi} \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}}$   
 $I = \int_0^{\pi} \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \int_0^{\pi} \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}}$
- 27. Problem based on reversing x from the integrand.  
 $I = \int_0^{\pi} \frac{dx}{\sin x} = \int_0^{\pi} \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \int_0^{\pi} \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}}$
- 28.  $I = \int_0^{\pi} \frac{dx}{\sin x} = \int_0^{\pi} \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \int_0^{\pi} \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}}$   
 $I = \int_0^{\pi} \frac{dx}{\sin x} = \int_0^{\pi} \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \int_0^{\pi} \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}}$
- 29.  $I = \int_0^{\pi} \frac{dx}{\sin x} = \int_0^{\pi} \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \int_0^{\pi} \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}}$   
 $I = \int_0^{\pi} \frac{dx}{\sin x} = \int_0^{\pi} \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \int_0^{\pi} \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}}$

- 25  $I = \int_0^{\pi} \frac{ax \cos x + b \sin x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$
- 26  $I = \int_0^{\pi} \frac{ax \cos x + b \sin x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$
- 27 Problem Based on Rearranging x from the integrand.
- 28  $I = \int_0^{\pi} \frac{ax \cos x + b \sin x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$
- 29  $I = \int_0^{\pi} \frac{ax \cos x + b \sin x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$
- 30  $I = \int_0^{\pi} \frac{ax \cos x + b \sin x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$

Adding (1) + (2)

$$I = \int_0^{\pi} \frac{x dx}{a^2 \cos^2 x + b^2 \sin^2 x} \text{--- (1)} ; I = \int_0^{\pi} \frac{(\pi - x) dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$2I = \int_0^{\pi} \frac{(x + \pi - x) dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$2I = \pi \int_0^{\pi} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

Prop:  $\int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx$  if  $f(2a-x) = f(x)$   
 = 0 if

- 26  $2I = \int_0^{\pi/2} dx = \dots$
- 27 Problem based on Riemann sum
- 28  $\int_0^{\pi/2} \cos x dx = \dots$
- 29  $2I = \int_0^{\pi/2} \frac{dx}{\cos^2 x} = \dots$
- 30  $I = \int_0^{\pi/2} \frac{dx}{\cos^2 x} = \dots$
- 31  $\int_0^{\pi/2} \frac{dx}{\cos^2 x} = \dots$

$$2I = \int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$I = \pi \int_0^{\pi/2} \frac{dx}{a^2 + b^2 \frac{\sin^2 x}{\cos^2 x}}$$

$$= \frac{\pi}{b^2} \int_0^{\pi/2} \frac{\sec^2 x dx}{\frac{a^2}{b^2} + \tan^2 x}$$

- 27. 0.11 hour  
Problem Based on Finding x  
from the integral.  
∫ dx / (a^2 + x^2) = tan^-1(x/a) + C
- 28. ∫ (x^2 + 1) dx = ∫ x^2 dx + ∫ 1 dx = (x^3/3) + x + C
- 29. ∫ (x^2 + 1) dx = (x^3/3) + x + C
- 30. ∫ (x^2 + 1) dx = (x^3/3) + x + C
- 31. ∫ (x^2 + 1) dx = (x^3/3) + x + C
- 32. ∫ (x^2 + 1) dx = (x^3/3) + x + C

Let  $\tan x = t$

$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$

$\sec^2 x dx = dt$

$x \rightarrow 0, \tan 0 \rightarrow 0 \Rightarrow t = 0$

$x \rightarrow \pi/2, \tan \pi/2 \rightarrow \infty \Rightarrow t = \infty$

$$I = \frac{\pi}{b^2} \int_0^\infty \frac{dt}{\left(\frac{a}{b}\right)^2 + t^2} = \frac{\pi}{b^2} \left[ \tan^{-1} \left( \frac{bt}{a} \right) \right]_0^\infty$$

$$= \frac{\pi}{ba} \left[ \tan^{-1}(\infty) - \tan^{-1}(0) \right] = \frac{\pi}{ba} \left[ \frac{\pi}{2} \right] = \frac{\pi^2}{2ab}$$